**William Doyle**

**November POM**

**Due 11/30/12**

# Making Change – November POM

## Step 1

**When I first looked at this problem with my group, I saw it would be a big undertaking. But I was not so much overwhelmed as aware that it would take a careful process to approach and potentially solve it. I noted that I would start with smaller bills to find how to make change for them under the given terms; and I hoped to find a pattern, in approaching small bills, that I could apply to larger bills such as 50, or any bill given!**

## Step 2

**Since I was planning to approach this problem by finding the number of ways to make change for small bills, my group and I initially decided to use a simple table like this:**

|  |  |
| --- | --- |
| **Value of Bill in Question** | **# of Ways to Make Change** |
| **1** | **0** |
| **2** | **1** |
| **3** | **2** |
| **4** | **…** |
| **5** | **…** |

**in which we ignored the bill itself as a way to make change for it. I would have then put a subscript beside each number to refer to the place in my work where it can be found.**

## Step 3

**For places in which I made mistakes, I will explain them here with superscripts to refer to where in the work process they occurred. It may be helpful to read the process first and come to these when they are relevant, since they reference my technique, perhaps strange to understand when not explained fully.**

**1Here I almost made a mistake that would have thrown off my results and patterns. When noting that one way to make the $11 dollar bill was taking a 2 and a 9, then finding all the ways to break up the 9 without using a 1, I noted that there were 7 ways to make 9 without a 1. So I added 7 to the current total number of ways to break up 11, neglecting the fact that there was also the 2 and 9 combination itself, meaning I should have added 8. Luckily, I caught this while looking back at my notes the next day.**

**2When calculating the number of ways to make 10 without a 1 (because I was addressing the $12 bill), I used the method described under “5”, but when referencing these notes, I subtracted the number of ways to make 10 without a 1 from the total number of ways to make 10, instead of subtracting the number of ways to make 10 with a 1 from the total, which would have given me the number of ways to make 10 without a 1, which is what I was looking for. I think I recognized this mistake pretty soon after I made it, but still, had I not caught it, it would have tainted my data.**

**I think it is safe to say that these errors were careless mistakes caused by going to quickly when not mentally focused enough.**

## Step 4

**I was talking to and working with Anja and Aidan, and Anja suggested this more complex, self-referential type of data table, which would help us find the ways to make the smaller bills by looking back at other bills in the table.**

# of combos

|  |  |  |  |
| --- | --- | --- | --- |
| **Bill in Question** | **Combinations (written out)** | **w/ bill** | **w/o bill** |
| **1** | **1** |  |  |
| **2** | **1+1, 2** |  |  |
| **…** | **…** |  |  |

**In this table, we wrote out all the combinations so that we could then refer to them with breaking up larger bills to make sure we weren’t repeating combinations in a different order. The “w/ bill” column would be used to find out how many ways there are to make that bill if you are counting the bill itself as a way to make change. For example, you might say 36 + 4 is a way to make 40, and by looking at the “w/ bill” column, you would know how many ways you can make the 4, so how many ways to make 40 with a $36 bill. (We soon realized this was not a very useful column since it values would obviously be the “w/o bill” column plus 1). The “w/o bill” column would be used to find patterns that we could apply to the $50 bill since essentially when addressing the it we wouldn’t count the 50 itself as a way to make change for a 50.**

**It was evident that writing out all the combinations would be tedious and long, but we were willing to do it for the first few. After that, we had no intention of still writing out the combinations. And when I found interesting ways to expedite the process of finding the number of combinations, this column simply became where I referenced my exterior notes on the bill in question.**

**When working with Anja and Aidan I checked my original data in which I had gathered info for bills 1-5, and they confirmed it.**

## Step 5

**11/5**

**When I first tried to make change for small bills, I used a sort of branching method with my group in class:**

**Bill = 1 = 0 ways to make it. Bill = 2 1 way to make 2**

1

1

**Bill = 3**

And so forth until 5. Essentially, the number of rows to the ‘tree’ was the number of ways to break it up. For numbers like 4 and 5 in which there were obviously more than 1 way to initially break the bill up (4 can be 3+1 or 2+2) I simply used multiple trees and made sure not to count repeats between trees.

2 ways to make 3

1

1

1

1

2

**I recorded my table the following numbers of combinations for bills 1-5, w/o bill:**

**1 = 0**

**2 = 1**

**3 = 2**

**4 = 4**

**5 = 6**

**Needless to say, this method showed itself to get very complicated very quickly. So we looked for another way to find the number of ways to make change.**

**11/6**

**We (my class group) saw that the most basic way to make change was to make the bill all out of 1’s. So we tried to start from a row of 1’s and do an ‘upside-down tree’ of combining them. For example, to find the ways to make change for a $6 bill, we would start with a row of 6 1’s:**

**1 1 1 1 1 1**

**Then combine them all into 2’s and combine those etc until the bill itself has been reached. Immediately we realized that not only would this be no less time consuming or complicated than the other method, but it would not account for ways of making the 6 bill that would be, say, one 2 and 4 1’s. So we kept looking for another way to find combinations.**

**11/10**

**1. Having gotten combinations for bills 1-5, I was working with Aidan and Anja to find a better**

**(more efficient) way to collect data. We made a discovery that we kept making improvements upon and which was our final way of collecting data. To make change for a bill, in this example, 6, we would take:**

**1 + 5 (1 basic way to make 6) – there are 6 other ways to make a “5” (not including the 5 itself), so therefore we now have 7 ways to make the $6 bill with a 1 and some combination of other bills that add up to 5.**

**Then you take 2 + 4, and note that only one way to make the 4 (2 + 2) has not been included in our ways to make 6 with 1 + 5. Therefore you add 2 to our current total, pushing the number of combinations for 6 up to 9.**

**Finally you take 3 + 3. There are no ways to break up this combination this that are not already included in our total. So you add 1 to the total, making it 10. 10 ways to make a 6.**

**2. For 7 (with same process)**

**1 + 6 – 10 other ways to make 6 = 11**

**2 + 5 – 1 other way to make 5 without 1 = 13**

**3 + 4 – 0 other ways = 14**

**NOTE: The reason I say “1 other way to make 5 without 1” is because we realized that the reason certain combinations were repeated is that if you chose, in this example, a way to make 5 that has a 1 in it, then the rest of the numbers in that combination will add up to six, meaning it is just a subdivision of the “1 + 6” unit which you have already addressed. The same applies for 3 + 4, for example. You must find ways to make 4 that have neither a 1 nor a 2 in them, to avoid repeating a combination. There are none, hence the “0 other ways”.**

**11/16**

**3. For 8:**

**1 + 7 – 14 other ways to make 7 = 15**

**2 + 6 – 3 other way to make 6 without 1 = 19**

**3 + 5 – 0 other ways = 20**

**4 + 4 – 0 other ways = 21**

**4. For 9:**

**1 + 8 – 21 other ways to make 8 = 22**

**2 + 7 – 3 other way to make 7 without 1 = 26**

**3 + 6 – 1 other way to make 6 without 1 or 2 = 28**

**4 + 5 – 0 other ways = 29**

**5. For 10:**

**1 + 9 – 29 other ways to make 9 = 30**

**2 + 8 – 6 other ways to make 8 without 1 = 37**

**3 + 7 – 1 other way to make 7 without 1 or 2 = 39**

**4 + 6 – 0 other ways = 40**

**5 + 5 – 0 other ways = 41**

**NOTE: Here I noticed another short cut. It was becoming tedious to try to find how many ways to make a given bill without a 1, or 1&2, etc. Then I noticed that to calculate this, all I have to do is go back to my notes on the given bill, and subtract the number of ways to make the bill with a 1 from the total number of ways to make the bill, giving me the number of ways to make the bill without a 1. A similar technique is employed for breaking up bills when we must avoid 1&2, 1,2&3, etc.**

**11/18**

**We (Aidan, Anja, Kiki and I) looked for patterns in the number of combinations for bills 1-10, confident that we had enough data (see scrap notes for 11/18). It quickly became evident that there was not one, for a pattern in the first difference between numbers of combinations repeated a few times and then stopped being applicable.**

**So Aidan proposed that we look at odd-numbered bills and even-numbered bills separately, since even-numbered bills always had that extra combination of two identical numbers. See my notes for this section to see the pattern that we saw might be occurring. In the odds, a constant 4th difference of 1 seemed to be possible. In the evens, a constant 4th difference of 2 seemed plausible. But we could certainly not be remotely convinced until we got more data.**

**11/22**

**I continued gathering data for 11 and 12, and checked the patterns.**

**16. For 11:**

**1 + 10 – 41 other ways to make 10 = 42**

**2 + 9 – 7 other ways to make 9 without 1 = 50**

**3 + 8 – 2 other ways to make 8 without 1 or 2 = 53**

**4 + 7 – 0 other ways w/o 1,2,3 = 54**

**5 + 6 – 0 other ways w/o 1-4 = 55**

**(superscript 1 for reference to error)**

**27. For 12:**

**1 + 11 – 55 other ways to make 11 = 56**

**2 + 10 – 11 other ways to make 10 without 1 = 68**

**3 + 9 – 3 other ways to make 9 without 1 or 2 = 72**

**4 + 8 – 1 other way w/o 1,2,3 = 74**

**5 + 7 – 0 other ways w/o 1-4 = 75**

**6 + 6 – 0 other ways w/o 1-5 = 76**

**(superscript 2 for reference to error)**

**I put these in my data table and checked to see if they continued the pattern I had in mind. They both fit perfectly with the respective constant 4th differences!!**

**11/25**

**I went on to try to further explore the pattern, to see if it would work with a 13 and 14 dollar bill:**

**8. For 13:**

**1 + 12 – 76 other ways to make 12 = 77**

**2 + 11 – 13 other ways to make 11 without 1 = 91**

**3 + 10 – 4 other ways to make 10 without 1 or 2 = 96**

**4 + 9 – 1 other way w/o 1,2,3 = 98**

**5 + 8 – 0 other ways w/o 1-4 = 99**

**6 + 7 – 0 other ways w/o 1-5 = 100**

**9. For 14:**

**1 + 13 – 100 other ways to make 13 = 101**

**2 + 12 – 20 other ways to make 12 without 1 = 122**

**3 + 11 – 5 other ways to make 11 without 1 or 2 = 128**

**4 + 10 – 2 other way w/o 1,2,3 = 131**

**5 + 9 – 0 other ways w/o 1-4 = 132**

**6 + 8 – 0 other ways w/o 1-5 = 133**

**7 + 7 – 0 other ways w/o 1-6 = 134**

**I again checked these values with my hypothesized patterns. The 13 bill DID NOT FIT with the odd-numbered pattern, but the 14 bill DID FIT with the even-numbered pattern. If I had more time, I would explore this potential pattern more. It is interesting that the even pattern is working longer than the odd pattern. But the fact that the odd pattern has failed has left me dubious about the other one. In any case, the search for this pattern was fruitful and interesting.**

### Step 6

**See attached paper**

#### **Resulting data table: see attached paper**

**Thoughts:**

**This problem of the month excited me the most of the 3 that we have had so far. I think it was something about the knowingly-futile attempt to find a pattern, and the excitement when one seemed to arise. But it was very enjoyable and interesting to go through the process, continuously improving on my method for gathering/recording data. I think perhaps the way in which it has helped me grow is that now I have a greater appreciation for the problem-solving process; I realize that it is so rewarding, when you try one method of approaching the problem and it doesn’t work, so you change it and try again, until finally you have something that works for you!**